Examiners’ Report

Principal Examiner Feedback

Summer 2017

Pearson Edexcel International A Level in Further Pure Mathematics (WFM03/01)

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General Introduction

Students found this paper difficult overall, hence the lower than usual grade boundaries. The paper started well for most but the integration and differentiation questions along with question 6 (b) proved to be particularly challenging.

Presentation was mostly good but there were cases of very muddled responses, possibly due to students' muddled thinking. This was particularly evident in question 6 (b) where students often wrote down everything they thought might be relevant in the hope that inspiration might strike - frequently it did not!

Students should be reminded that at this level almost all "show" questions need a conclusion to indicate that the student is aware that the the work is complete. Also in such questions every step, no matter how simple, must be included. Examiners cannot read students' minds - they can only mark what is written on the page.

Reports on Individual Questions

Question 1

This was a good straightforward question to start the examination. There were very few errors and most students gained full marks.

All but a very small minority used the correct exponential forms for cosh and sinh. A small number of students made algebraic slips when substituting into the given equation and collecting terms. Of those who simplified to a quadratic equation the vast majority solved by using factorisation with only a small minority attempting the quadratic formula. A small number of students gave one of their answers as – ln3 which was not in the required form and lost the final mark.

Question 2

This question was answered well with a high proportion of students scoring full marks.

In part (a) a few students were unclear that the "T" required them to write down the transpose matrix and a search for cofactors or a random change of sign was seen.

Multiplication of matrices **A** and **B** in the correct order was mostly correct in part (b). Accuracy was essential as a slip in just one entry lead to the loss of all three accuracy marks in this question.

In part (c) most students wrote down the matrix $(\mathbf{AB})^T$ before calculating $\mathbf{B}^T\mathbf{A}^T$. A number of students clearly rewrote their answer to $(\mathbf{AB})^T$ without checking the arithmetic for $\mathbf{B}^T\mathbf{A}^T$. This was fine if the previous answer was correct but an honest reworking may well have revealed an earlier slip. Students did not always realise the need for a concluding statement that the two parts $(\mathbf{AB})^T$ and $\mathbf{B}^T\ \mathbf{A}^T$ were equal.

Question 3

This question proved to be a challenging one for a lot of students. The need to use the chain rule and either the quotient or product rule in part (a) was usually understood though some students did forget to include one of the two elements. Manipulating the details, including keeping track of the signs, was often done badly. Very few students attempted the alternative methods.

Those who scored all the marks in part (a) usually went on to answer part (b) correctly. A small number who had made no progress in part (a) still attempted part (b) using k with some success. However, only 2 of the 4 marks for part (b) were available to students who did not use a numerical value of k which they had obtained in part (a).

Question 4

This question provided a valuable source of marks for most students. The setting up of the characteristic equation in part (a) was done well with only a few errors seen. Most students then chose to rearrange the characteristic equation into a cubic equation and it was this step that caused problems as algebraic errors stopped the production of the correct cubic which then did not factorise or have 6 as a root.
A few students used the factor theorem to verify the 6 as a root separately with most choosing to go straight for the factorisation of the cubic producing all three roots at once. Students who did not occasionally failed to verify that 6 is an eigenvalue. Those who used long division often made mistakes; this was particularly disappointing, especially from Further Maths students.

In part (b) the method of finding an eigenvector was well known. Having found an eigenvector a minority of students did not proceed to normalise it.

Question 5

Proof of the reduction formula in part (a) was challenging for a very high proportion of the students and many failed to score any marks here.

Quite a few attempted to split the expression $\text{cosec}^{n-2}x$ as $\text{cosec}^{n-1}x\,\text{cosec}x$ and integrate by parts. They soon realised that further progress was impossible.

Many students identified a suitable split as $\text{cosec}^{n-2}x\,\text{cosec}^2x$ and proceeded to integrate by parts. Lack of a starting formula or specific reference to the four parts $u, v, \frac{\mathrm{d}u}{\mathrm{d}x}$ and $\frac{\mathrm{d}v}{\mathrm{d}x}$ (or f, g, f' and g') meant it was difficult to be sure a correct method was employed. Three negative quantities were involved and many solutions tried to simplify before writing down the complete expression. Many students realised the need to replace $\cot^2 x$ with $\text{cosec}^2x - 1$ though sign errors in the formula were not unusual. Students reaching this point generally recognised how I_n and I_{n-2} appeared and further progress was made.

An alternative strategy favoured by a number of students was to replace $\text{cosec}^{n-2}x\,\text{cosec}^2x$ with $\text{cosec}^{n-2}x\left(1+\cot^2 x\right)$. Few following this method were then able to split up and integrate by parts.

In part (b) most student managed to score marks for the evaluation of I_4 and there were a large number of correct solutions. Few errors were made in applying the reduction formula to express I_4 in terms of I_2. Evaluation of I_2 was well done when approached as $\int \text{cosec}^2x\,\mathrm{d}x$ though a few solutions using the reduction formula for a second time thought that $\int \text{cosec}^0x\,\mathrm{d}x$ was needed when it should have been multiplied by zero. The formula

$\cot^2 x = \text{cosec}^2 x - 1$ was occasionally incorrect or not used .A few solutions left $-\frac{2}{3}\cot x - \frac{1}{3}\cot x$ in the final answer.

Question 6

Part (a) was answered well with most students scoring full marks. Few errors were made differentiating implicitly to reach a gradient of $\frac{b\sec^2\theta}{a\sec\theta\tan\theta}$ or $\frac{b}{a\sin\theta}$. Most students applied the formula $y - y_1 = m(x - x_1)$ to write down the equation of the tangent and then rearranged using $\sec^2\theta - \tan^2\theta = 1$ to reach the printed answer. There were occasional careless errors. A few solutions preferred the $y = mx + c$ approach to produce a tangent equation though this did require a lot more manipulation.

Part (b) proved to be a major challenge for students and few correct solutions were seen. The approach for many seemed to be to write down any formulae they knew and then play around without any obvious coherent strategy. A simple diagram may well have helped understand the overall direction in which to proceed. Often a maximum of two marks were scored by writing the focus as $(ae,0)$ and applying these coordinates in the tangent equation to obtain $e\sec\theta = 1$ or $\cos\theta = e$. Occasionally $\pm ae$ was written down and it was quite a search to see that only the positive one was being used.

There were few errors writing down a formula for the eccentricity of the ellipse though it was not always applied in a convincing way. The easiest way to the answer was to write the answer to the gradient from part (a) as $\frac{b}{a\sin\theta} = \frac{b}{a\sqrt{1-\cos^2\theta}} = \frac{b}{a\sqrt{1-e^2}}$. The application of the eccentricity formula soon reaches a gradient of value 1. Solutions which used the eccentricity formula at an earlier stage to reach $\sec\theta = \frac{a}{\sqrt{a^2 - b^2}}$ often stopped as $\tan\theta$ was not calculated. The approach in which the equation of the tangent was simplified to $y = x - \sqrt{a^2 - b^2}$ was the least popular, but when done this way was generally successful.

Question 7

This proved a challenging question for many students. In part (a) the idea of splitting the integrand into two separate terms was not well known and so students achieved very few marks for the whole question. Of those who did split the integrand there were many perfect solutions seen with only a minority having problems with the various powers and square root terms. The alternative substitution method was rarely seen but of those who chose this route there were many completely correct solutions seen. Some tried to integrate by parts and were unable to progress very far. Arcsin was usually obtained by students with the odd slip up with either p or q. The other part of the integral was nearly always reached but a minority of students gave a function of ln rather than a square root.

Part (b) was dependent on part (a). Most students used a sound method but errors in part (a) led to a failure to gain the A marks here.

Question 8

In part (a), students could apply the formula for the required surface area producing a majority of perfectly correct solutions.

However part (b) proved very difficult with very few correct solutions seen. Indeed there were a lot of students who left this part blank which may have been due to the lack of time. Those who chose to express the integrand in terms of a sine function of the half angle could then usually proceed to a correct answer.
There were attempts at solutions seen in which a calculator had clearly been used to get the area as no working was shown and these gained no credit. Many started part (b) using a substitution but mainly substituting incorrectly. Very few substituted $u = 1 - \cos\theta$ and those who did generally failed to achieve an expression in terms of u only as they did not replace $\mathrm{d}\theta$ correctly. A novel method used once that substitution had been done was to rewrite the numerator as $-(2 - u - 2)$ and then proceed to split the integrand. This avoided the use of integration by parts. Very few students brought part (b) to a correct conclusion.

Question 9

This was an accessible question with many students scoring full marks for parts (a) and (b), but part (c) was less successfully attempted.

For part (a) the most common error was to fail to use 3 edges with a common point. A few students used a vertex rather than an edge, often using two edges in their cross product but then the position vector of a vertex for the scalar product. There were occasional errors involving the 1/6, with either the 1/6 being omitted altogether or 1/3 being used instead.

The required method was well known in part (b). Errors in finding a normal vector often followed from errors in part (a). A small number of students gave their answer in the wrong form, possibly not reading the question carefully enough or possibly indicating a lack of understanding of the difference between the vector equation, Cartesian equation and parametric equations for the plane.

In part (c) it was disappointing to see that many students attempted to use incorrect methods and scored no marks. Only a minority of students were able to produce a correct parametric expression for a point on *DT* to enable them to find the value of the parameter and hence the coordinates of the required point. Those who knew what they were doing generally scored full marks, with only occasional numerical slips along the way.

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